Coulomb's law

We know that a field exists around a charge that exerts force on other charges placed there, but how can we calculate the force? The force will be dependent upon the sizes of the charges, and their separation. In fact the force follows an inverse square law, and is very similar in form to Newton's Law of Universal Gravitation. It is known as Coulomb's law, and it is expressed as:

$$F = \frac{kQ_1Q_2}{r^2}$$

where F = force on each charge (N)

Q_1 and Q_2 are the interacting charges (C)

r = separation of the charges (m)

The k is a constant of proportionality (like G in Newton's Law of Universal Gravitation). In a vacuum, and to all intents and purposes, in air, we have

$k = 9.0 \times 10^9$ N m^2 C^{-2} (units obtained by rearranging the original equation)

More traditionally, Coulomb's law is written:

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$

where ε_0 is known as the "permittivity of free space"; $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} (farads per metre). Permittivity is a property of a material that is indicative of how well it supports an electric field, but is beyond the scope of these notes. Thus, we have $k = 1/(4\pi\,\varepsilon_0)$. Different materials have different permittivities, and so the value of k in Coulomb's law also changes for different materials.

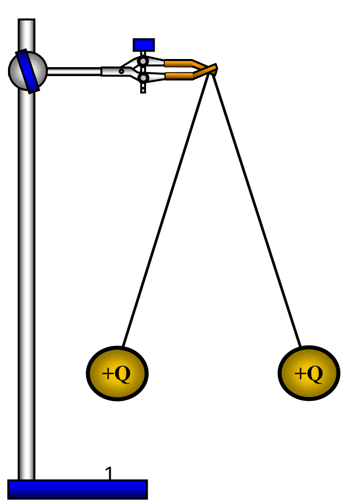
Points to bring out about Coulomb's law:

The form is exactly the same as Newton's Law of Universal Gravitation; in particular, it is an inverse-square law.

This force can be attractive or repulsive. The magnitude of the force can be calculated by this equation, and the direction should be obvious from the signs of the interacting charges. (Actually, if you include the signs of the charges in the equation, then whenever you get a negative answer for the force, there is an attraction, whereas a positive answer indicates repulsion).

Although the law is formulated for point charges, it works equally well for spherically symmetric charge distributions. In the case of a sphere of charge, calculations are done assuming all the charge is at the centre of the sphere.

In all realistic cases, the electric force between 2 charges objects absolutely dwarfs the gravitational force between them, as the first of the worked examples will show.



(resourcefulphysics.org)

Coulomb's law

Data required:

$k = 9.0 \times 10^{9}$ N m^2 C^{-2}

mass of an electron = 9.11×10^{-31} kg

mass of a proton = 1.67×10^{-27} kg

G = 6.67×10^{-11} N m^2 kg^{-2}

1. What is the force of repulsion between two electrons held one metre apart in a vacuum? What is the gravitational force of attraction between them? By what factor is the electric repulsion greater than the gravitational attraction?

2. By what factor is the electric force between two protons greater than the gravitational force between them?

3. Given the difference in magnitudes of gravitational and electrical forces you've just discovered, why do you feel gravitational attraction from the earth, but no electrical forces?

Answers and worked solutions

1. $F = kQ_1Q_2/r^2 = (9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19})/ 1^2 = 2.3 \times 10^{-28}$ N

   $F = Gm_1m_2/r^2 = (6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 9.11 \times 10^{-31})/ 1^2 = 5.5 \times 10^{-71}$ N

   Electrical force/gravitational force = $2.3 \times 10^{-28} / 5.5 \times 10^{-71} = 4 \times 10^{42}$!!!!

2. Electrical force is same as with 2 electrons (they have the same magnitude of charge).

   Gravitational force = $(6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27})/ 1^2 = 1.9 \times 10^{-64}$ N

   Electrical force/gravitational force = 1.2×10^{36}

3. Because even though the Earth has a very large mass, providing a strong gravitational field, both it and you are electrically neutral overall.